A Study on the Wide-Sense Stationarity and Mean Ergodicity of SOC Rayleigh Fading Channel Simulators

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Abstract. Stationarity and ergodicity are desirable properties of any stochastic simulation model for small-scale mobile radio channels. These properties enable the channel simulator to accurately emulate the channel's statistical properties in a single simulation run without requiring information on the time origin. In a previous paper, we analyzed the ergodicity with respect to (w.r.t.) the autocorrelation function (ACF) of seven fundamental classes of stochastic sum-of-cisoids (SOC) simulation models for mobile Rayleigh fading channels. In this paper, we continue our investigations on the subject by providing a comprehensive study on the wide-sense stationarity and the ergodicity w.r.t. the mean value of these classes of SOC channel simulators. The obtained results can be used in connection with those presented in our previous paper to design efficient channel simulators for the performance evaluation of modern mobile communication systems.

Keywords: Channel simulators, ergodic processes, mean value, mobile communications, sum-of-cisoids, wide-sense stationary processes.

1 Introduction

The proliferation of low-cost electronic devices with high computational capabilities and the need that exists among telecommunications engineers for affordable and powerful tools for the performance evaluation of modern mobile communication systems have turned the design of computer simulators into a major subject of research. When designing a simulator for the performance assessment of wireless communication systems, it is fundamental to choose a proper model to simulate the channel. This is of primary importance, since the channel exerts a strong influence on the system's performance [16, Ch. 15]. Several different simulation models for multipath radio channels have been proposed in the literature, such as those based on autoregressive processes [1], digital filters [17], linear transformations of complex Gaussian sequences [4], and Karhunen-Lòeve expansions of stochastic processes [19]. However, simulation models based on a finite sum of complex sinusoids (cisoids) have been shown to be an excellent basis for the design of single-input single-output (SISO) [7, 8] and multiple-input multiple-output (MIMO) [15, 20, 21] multipath radio channel simulators. Sum-of-cisoids (SOC) models are well suited for the simulation of fading channels under both isotropic and non-isotropic scattering conditions, as demonstrated in [5]. They have found applications, e.g., in the laboratory analysis of space-time coding schemes [18].

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Two desirable properties of any stochastic simulation model for small-scale multipath radio channels are stationarity and ergodicity. These properties enable the channel simulator to accurately emulate the channel's statistical properties in a single simulation run (ergodicity) without requiring information on the time origin (stationarity). In the strict sense, a channel simulator is stationary if all marginal and joint probability density functions (PDFs) of the random process characterizing the underlying simulation model are time independent. On the other hand, a channel simulator is ergodic if the time averages of the simulation model are equal to the ensemble averages. These conditions are too stringent and are hardly satisfied in practice. However, the information about the channel's third- or higher-order statistics is rarely required to assessing the performance of wireless communication systems. Hence, for most practical purposes, it suffices if the channel simulator is wide-sense stationary (WSS) and ergodic with respect to (w.r.t.) the mean value and the autocorrelation function (ACF). Indeed, an important part in the statistical characterization of a channel simulator consists in determining whether the simulation model is a WSS, a mean ergodic (ME), or/and an autocorrelation ergodic (AE) random process.

In a previous paper [6], we analyzed the autocorrelation ergodicity of seven fundamental classes of stochastic SOC simulation models for mobile Rayleigh fading channels. In this paper, we continue our investigations on the subject by providing a comprehensive study on the wide-sense stationarity and mean ergodicity of these classes of SOC channel simulators. To the best of the authors' knowledge, the WSS and ME properties of stochastic SOC channel simulators have not been systematically analyzed so far. We notice, nonetheless, that some partial results are available in the literature. In [5], the wide-sense stationarity, mean ergodicity, and autocorrelation ergodicity of a class of SOC models defined by cisoids with constant gains, constant frequencies, and random phases were studied. In [9], the first-order stationarity of the envelope of the seven fundamental classes of stochastic SOC models was investigated. The work in [12–14] is also worth mentioning. There, the authors analyzed the wide-sense stationarity, mean ergodicity, and autocorrelation ergodicity of stochastic sum-of-sinusoids simulation (SOS) for mobile fading channels. Despite the similarities between SOS and SOC models, we point out that the results obtained in [12–14] are not valid for SOC channel simulators. This is because the ACF of an SOC model has more degrees of freedom than the ACF of a conventional SOS model, as explained in [6, Sec. II]. The findings reported in this paper complement those presented in [5,6,9,12–14] and can be used as guidelines to design efficient channel simulators for the performance evaluation of modern mobile communication systems.

The outline to the rest of the paper is as follows. In Section 2, we provide a brief description of an statistical reference model for narrowband mobile Rayleigh fading channels. In Section 3, we review the characteristics of the seven classes of stochastic SOC simulation model for mobile Rayleigh fading channels. In Section 4, we systematically analyze the WSS and ME properties of the classes of stochastic SOC channel simulators. Finally, Section 5 concludes the paper with some remarks and a summary of key results. As a notational convention, we will use bold symbols and letters to denote random variables and stochastic processes, whereas normal symbols and letters will be used for constants and deterministic processes.

2 The Reference Model

A small-scale narrowband mobile Rayleigh fading channel can be represented in the equivalent baseband by a complex Gaussian random process

$$\boldsymbol{\mu}(t) = \boldsymbol{\mu}_I(t) + j\boldsymbol{\mu}_Q(t), \qquad j \triangleq \sqrt{-1} \tag{1}$$

where $\boldsymbol{\mu}_I(t)$ and $\boldsymbol{\mu}_Q(t)$ are stationary real-valued Gaussian processes with mean zero and variance $\sigma_{\boldsymbol{\mu}}^2/2$. Equation (1) may be rewritten in phasor notation as

$$\boldsymbol{\mu}(t) = \boldsymbol{\zeta}(t)\exp\{j\boldsymbol{\phi}(t)\} \tag{2}$$

where $\boldsymbol{\zeta}(t) = \sqrt{\boldsymbol{\mu}_I^2(t) + \boldsymbol{\mu}_Q^2(t)}$ and $\boldsymbol{\phi}(t) = \arctan\left(\boldsymbol{\mu}_Q(t)/\boldsymbol{\mu}_I(t)\right)$. One can easily verify that the first-order PDF of $\boldsymbol{\zeta}(t)$ equals the Rayleigh distribution with parameter $\sigma_{\boldsymbol{\mu}}$ [11, Sec. 5.5], while the first-order statistics of $\boldsymbol{\phi}(t)$ are characterized by a circular uniform PDF [11, Sec. 5.6].

The statistical properties of the Rayleigh fading channel model described by the complex Gaussian process in (1) are completely specified by the time-shift insensitive (TSI) ACF $r_{\boldsymbol{\mu\mu}}(\tau)$ of $\boldsymbol{\mu}(t)$, where $r_{\boldsymbol{xx}}(\tau) \triangleq E\{\boldsymbol{x}^*(t)\boldsymbol{x}(t+\tau)\}$ for an arbitrary random process $\boldsymbol{x}(t)$. The operators $E\{\cdot\}$ and $(\cdot)^*$ stand for the statistical expectation and the complex conjugate, respectively. Assuming a two-dimensional fixed-to-mobile propagation environment, the ACF of $\boldsymbol{\mu}(t)$ can be written as [3]

$$r_{\boldsymbol{\mu\mu}}(\tau) = \sigma_{\boldsymbol{\mu}}^2 \int_{-f_{\max}}^{f_{\max}} p_{\boldsymbol{f}}(f)\exp\left\{j2\pi f\tau\right\} df. \tag{3}$$

where $f_{\max}$ is the maximum Doppler shift caused by the movement of the mobile terminal, and $p_{\boldsymbol{f}}(f)$ is the PDF of the random Doppler frequencies of the channel's multipath components.

3 SOC Simulation Models

Most of the statistical properties of $\boldsymbol{\mu}(t)$ relevant for system performance analysis—such as its correlation properties, spectral characteristics, and the first-order distributions of its envelope and phase—can accurately be emulated via a simulation model based on a finite SOC, as demonstrated in [2, 7, 8]. Figure 1 shows the general structure of an SOC Rayleigh fading channel simulator with N homogeneous cisoids[1], the parameters of which—gains, frequencies, and phases—are defined either as random variables or deterministic quantities. An SOC simulation model can mathematically be described by a complex random process $\hat{\boldsymbol{\mu}}(t)$ if any of the cisoids' parameters is random, otherwise, it is to be represented by a complex deterministic process $\hat{\mu}(t)$. A classification of SOC channel simulators based on the type of the cisoids' parameters was introduced in [9]. All in all, eight fundamental classes of SOC simulators were identified in that paper.

[1] By homogeneous cisoids we mean a group of cisoids characterized by the same type of parameters.

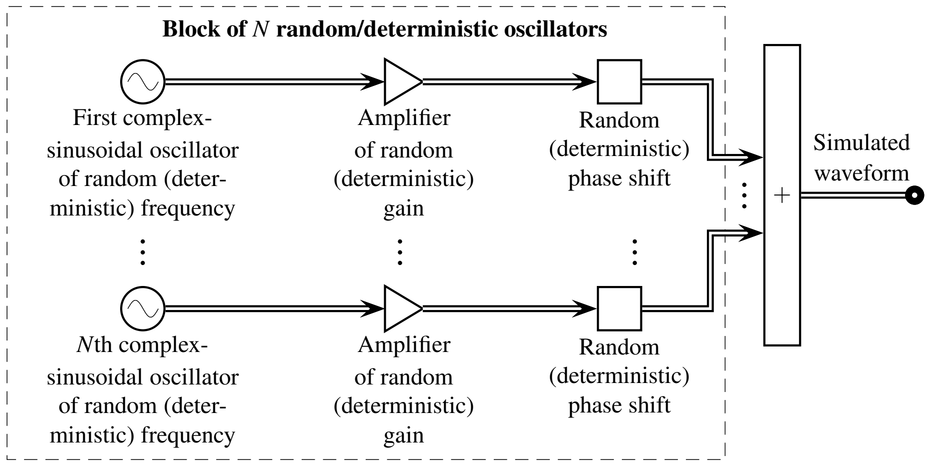

Fig. 1. Block diagram of an SOC simulation model for Rayleigh fading channels [6].

Table 1. Classification of SOC simulation models for Rayleigh fading channels with respect to the type of the cisoids' parameters [9].

Parameters	**Gains**	**Frequencies**	**Phases**
Class I	Deterministic	Deterministic	Deterministic
Class II	Deterministic	Deterministic	Random
Class III	Deterministic	Random	Deterministic
Class IV	Deterministic	Random	Random
Class V	Random	Deterministic	Deterministic
Class VI	Random	Deterministic	Random
Class VII	Random	Random	Deterministic
Class VIII	Random	Random	Random

These classes, which are listed in Table 1, will be taken as a reference in this paper to carry out our investigations on the WSS and ME properties of SOC channel simulators. For the analysis presented herein it is assumed that the following holds:

– All random variables are statistically independent.
– If the cisoids' phases are random variables, then they are uniformly distributed over $[-\pi, \pi)$.
– If the cisoids' Doppler frequencies are random variables, then they have a PDF $p_{\boldsymbol{f}}(f)$ identical to that characterizing the statistics of the reference model's Doppler frequencies.
– If the cisoids' gains are random variables, then they are identically distributed with a mean value m_c and a variance $\sigma_c^2 = \sigma_{\boldsymbol{\mu}}^2/N$.

The concepts of stationarity and ergodicity do not apply on the Class I simulators, since this class of models is completely deterministic. However, the information about the temporal mean value (TMV) of the Class I simulators is fundamental to find out whether or not a given class of stochastic SOC models is defined by a set of ME processes.

4 WSS and Mean Ergodic Properties of SOC Channel Simulators

4.1 Definitions

Before we proceed to analyze the WSS and ME properties of SOC simulation models, it is convenient to formally define the concepts of wide-sense stationarity and mean ergodicity.

Definition 1 (WSS process) *Let $\hat{\boldsymbol{\mu}}(t)$ be a random process. Then, $\hat{\boldsymbol{\mu}}(t)$ is said to be WSS if [10, p. 555]:*

- *The mean value $m_{\hat{\boldsymbol{\mu}}}(t) \triangleq E\{\hat{\boldsymbol{\mu}}(t)\}$ of $\hat{\boldsymbol{\mu}}(t)$ is time independent, i.e., $m_{\hat{\boldsymbol{\mu}}}(t) = m_{\hat{\boldsymbol{\mu}}}$.*
- *The ACF $r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(t_1,t_2) \triangleq E\{\hat{\boldsymbol{\mu}}^*(t_1)\hat{\boldsymbol{\mu}}(t_2)\}$ of $\hat{\boldsymbol{\mu}}(t)$ depends only on the time difference $\tau = t_2 - t_1$, meaning that $r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(t_1,t_2)$ is TSI, so that $r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(t_1,t_2) = r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(\tau)$.*

Definition 2 (ME process) *Let $\hat{\boldsymbol{\mu}}(t)$ be a random process whose mean value $m_{\hat{\boldsymbol{\mu}}}(t) \triangleq E\{\hat{\boldsymbol{\mu}}(t)\}$ is constant over time, so that $m_{\hat{\boldsymbol{\mu}}}(t) = m_{\hat{\boldsymbol{\mu}}}$. Then, $\hat{\boldsymbol{\mu}}(t)$ is said to be ME if [10, Sec. 6.6]:*

- *The TMV $m_{\hat{\mu}}^{(k)} \triangleq \langle \hat{\mu}^{(k)}(t) \rangle$ of every sample function $\hat{\mu}^{(k)}(t)$ of $\hat{\boldsymbol{\mu}}(t)$ is equal to $m_{\hat{\boldsymbol{\mu}}}$, i.e., $m_{\hat{\mu}}^{(k)} = m_{\hat{\boldsymbol{\mu}}}\ \forall k$.*

The notation $\langle x(t) \rangle$ stands for the time average of an arbitrary function of time $x(t)$.

4.2 Classes of SOC Channel Simulators and Their Autocorrelation Properties

For the analysis of the wide-sense stationarity, it is necessary to determine if the ACF of the random process $\hat{\boldsymbol{\mu}}(t)$ characterizing each of the seven classes of stochastic SOC models is a TSI function. This was already done in [6]. The results there obtained are summarized in Table 2, where θ_n denotes the phase of the n-th cisoid. This table also summarizes the conclusions drawn in [6] regarding the autocorrelation ergodicity of the seven classes of stochastic SOC models.

4.3 Classes of SOC Channel Simulators and Their WSS and ME Properties

Class I Channel Simulators The simulation models of Class I are characterized by a deterministic SOC model

$$\hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp\left\{ j(2\pi f_n t + \theta_n) \right\} \tag{4}$$

where the cisoids' gains c_n, Doppler frequencies f_n, and phases θ_n are arbitrary constants. To ensure that the Doppler power spectral density (DPSD) of $\hat{\mu}(t)$ is band-limited, it is assumed that $f_n \in (-f_{\max}, f_{\max})$, $\forall n$. The TMV $m_{\hat{\mu}} \triangleq \langle \hat{\mu}(t) \rangle$ of this class of deterministic SOC models is given as

$$m_{\hat{\mu}} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{\mu}(t)dt. \tag{5}$$

Table 2. Classes of SOC models and their autocorrelation properties [6].

Class	Gains	Frequencies	Phases	AE	TSI ACF
I	Deterministic	Deterministic	Deterministic	—	—
II	Deterministic	Deterministic	Random	Yes	Yes
III	Deterministic	Random	Deterministic	No	No/Yes[a]
IV	Deterministic	Random	Random	No	Yes
V	Random	Deterministic	Deterministic	No	No/Yes[b]
VI	Random	Deterministic	Random	No	Yes
VII	Random	Random	Deterministic	No	No/Yes[b]
VIII	Random	Random	Random	No	Yes

[a] If the boundary condition $\sum_{n=1}^{N}\sum_{m=1,m\neq n}^{N}\exp\{j(\theta_m-\theta_n)\}=0$ is satisfied.

[b] If the mean value of the random gains is equal to zero.

Substituting (4) in (5), and assuming that $f_n \neq 0$, $\forall n$, we have

$$m_{\hat{\mu}} = 0. \tag{6}$$

The k-th sample function of an stochastic SOC model can be represented by a deterministic process $\hat{\mu}^{(k)}(t)$ similar to that defined in (4). Therefore, the TMV $m_{\hat{\mu}^{(k)}} \triangleq \left\langle \hat{\mu}^{(k)}(t) \right\rangle$ of $\hat{\mu}^{(k)}(t)$ is given as in (6) for all k regardless of the class of stochastic SOC models under consideration.

Class II Channel Simulators Simulation models of this class are characterized by a stochastic SOC model of the form

$$\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} c_n \exp\left\{ j(2\pi f_n t + \boldsymbol{\theta}_n)\right\}. \tag{7}$$

It is straightforward to verify that

$$m_{\hat{\boldsymbol{\mu}}}(t) = E\{\hat{\boldsymbol{\mu}}(t)\} = 0. \tag{8}$$

With reference to Definition 2, and taking account of (6) and (8), we can conclude that the SOC models of Class II are ME, since $m_{\hat{\mu}}^{(k)} = m_{\hat{\boldsymbol{\mu}}} = 0$, $\forall k$. Furthermore, given that $m_{\hat{\boldsymbol{\mu}}}(t)$ is constant over time and the ACF of $\hat{\boldsymbol{\mu}}(t)$ is TSI (see Table 2), it follows that the Class II SOC simulators are WSS processes.

Class III Channel Simulators This class of simulators is defined by the set of stochastic processes of the form $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} c_n \exp\left\{ j(2\pi \boldsymbol{f}_n t + \theta_n)\right\}$. For this class of SOC simulators, we have

$$m_{\hat{\boldsymbol{\mu}}}(t) = \frac{r_{\boldsymbol{\mu\mu}}(t)}{\sigma_{\hat{\boldsymbol{\mu}}}^2} \sum_{n=1}^{N} c_n \exp\{j\theta_n\} \tag{9}$$

where $r_{\boldsymbol{\mu\mu}}(t)$ is the TSI ACF of the reference channel model [see (3)]. Since the mean value of $\hat{\boldsymbol{\mu}}(t)$ is time dependent, it follows that the Class III SOC simulators are not WSS or ME processes. However, $\hat{\boldsymbol{\mu}}(t)$ proves to be a ME process if any of the following conditions is met:

- The number of cisoids N is even and the phases θ_n are given such that $\theta_n = -\theta_{n+N/2} = \pi/2$ for $n = 1,\ldots,N/2$.
- The number of cisoids N is even, the phases θ_n are equal to each other, and the gains are given such that $c_n = -c_{n+N/2}$ for $n = 1,\ldots,N/2$.

In turn, the wide-sense stationarity property of $\hat{\boldsymbol{\mu}}(t)$ holds if any of the aforementioned conditions is fulfilled and $\sum_{n=1}^{N}\sum_{m=1,m\neq n}^{N}\exp\{j(\theta_m - \theta_n)\} = 0$ (cf. Table 2).

Class IV Channel Simulators The Class IV simulators are characterized by a stochastic process $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} c_n \exp\left\{j(2\pi \boldsymbol{f}_n t + \boldsymbol{\theta}_n)\right\}$. It is straightforward to show that the mean value of the Class IV simulators is time independent and equal to $m_{\hat{\boldsymbol{\mu}}} = 0$. We can therefore conclude that the Class IV of SOC simulators is defined by a set of ME random processes. Moreover, since the ACF of this class of SOC simulation models is TSI (see Table 2), it follows that the SOC models of this class are also WSS random processes.

Class V Channel Simulators This class of simulators is defined by the set of stochastic SOC models $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} \boldsymbol{c}_n \exp\left\{j(2\pi f_n t + \theta_n)\right\}$. In this case, the mean value of $\hat{\boldsymbol{\mu}}(t)$ is equal to

$$m_{\hat{\boldsymbol{\mu}}}(t) = m_c \sum_{n=1}^{N} \exp\{j(2\pi f_n t + \theta_n)\}. \tag{10}$$

It is clear from the previous equation that if $m_c \neq 0$, then the mean value of the Class V simulators is time dependent and $\hat{\boldsymbol{\mu}}(t)$ is not a WSS nor a ME process. However, if the mean value of the random gains $\boldsymbol{c}_n$ is equal to zero, then $m_{\hat{\boldsymbol{\mu}}}(t) = 0$. Thereby, $\hat{\boldsymbol{\mu}}(t)$ proves to be a WSS and a ME random process, since $m_{\hat{\mu}(k)} = m_{\hat{\boldsymbol{\mu}}}$ for all k, and $r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(t_2,t_2) = r_{\hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}}(\tau)$ if $m_c = 0$ (cf. Table 2).

Class VI Channel Simulators The Class VI simulators are characterized by a stochastic process $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} \boldsymbol{c}_n \exp\left\{j(2\pi f_n t + \boldsymbol{\theta}_n)\right\}$. For this class of simulators, we have that $m_{\hat{\boldsymbol{\mu}}}(t) = m_{\hat{\boldsymbol{\mu}}} = 0$. The simulation models of Class VI are therefore ME processes. Moreover, they are also WSS, since their ACF is always TSI (cf. Table 2).

Class VII Channel Simulators This class of simulators is defined by the set of stochastic SOC models of the form $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} \boldsymbol{c}_n \exp\left\{j(2\pi \boldsymbol{f}_n t + \theta_n)\right\}$. The mean value of this class of simulators is equal to

$$m_{\hat{\boldsymbol{\mu}}}(t) = \frac{m_c\, r_{\boldsymbol{\mu\mu}}(t)}{\sigma_{\boldsymbol{\mu}}^2} \sum_{n=1}^{N} \exp\{j\theta_n\}. \tag{11}$$

From the previous equation, it follows that the mean value of the Class VII simulators is time independent if $m_c = 0$ or if the number of cisoids is even and $\theta_n = -\theta_{n+N/2} = \pi/2$ for $n = 1, \ldots, N/2$. If any of these conditions is fulfilled, then the SOC simulators of Class VII are ME processes. On the other hand, based on the results presented in Table 2, we can conclude that the SOC simulators of this class are both ME and WSS if and only if $m_c = 0$.

Class VIII Channel Simulators Simulation models of the Class VIII are characterized by a random process $\hat{\boldsymbol{\mu}}(t) = \sum_{n=1}^{N} \boldsymbol{c}_n \exp\left\{j(2\pi \boldsymbol{f}_n t + \boldsymbol{\theta}_n)\right\}$. For this class of SOC models, one can easily verify that $m_{\hat{\boldsymbol{\mu}}}(t) = m_{\hat{\boldsymbol{\mu}}} = 0$. In view of this result, we can conclude that the Class VIII SOC simulation models are ME random processes. They are also WSS processes, as it was found in [6] that the ACF of this class of simulators is a TSI function.

5 Conclusions

In this paper, we continued our investigations on the stationarity and ergodicity of SOC simulation models for mobile Rayleigh fading channels. Specifically, we analyzed the WSS and ME properties of seven fundamental classes of stochastic SOC channel simulators. Based on the results presented in this paper, we can conclude that the SOC simulators of classes II, IV, VI, and VIII are always WSS and ME random processes. On the other hand, SOC simulators of classes III, V, and VII are WSS and ME provided that some specific conditions are fulfilled. The findings reported herein complement those presented in a previous paper, where we analyzed the AE properties of the seven classes of stochastic SOC models. Table 3 summarizes the results obtained in both papers. As a final remark, we observe that only the simulation models of Class II possesses the desired WSS, ME, and AE properties. Hence, this type of models provide an excellent basis for the design of efficient channel simulators for the performance analysis of mobile communication systems.

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References

1. Baddour, K.E., Beaulieu, N.C.: Autoregressive modeling for fading channel simulation. IEEE Trans. Wireless Commun. 4(4), 1650–1662 (Jul 2005)
2. Cheng, X., Wang, C.X., Laurenson, D.I., Salous, S., Vasilakos, A.V.: New deterministic and stochastic simulation models for non-isotropic scattering mobile-to-mobile rayleigh fading channels. Wirel. Commun. Mob. Comput. (Oct 2009), DOI: 10.1002/wcm.864
3. Clarke, R.H.: A statistical theory of mobile radio reception. Bell Syst. Tech. J. 47, 957–1000 (Jul 1968)

Table 3. Classes of SOC Rayleigh fading channel simulation models and their stationary and ergodic properties.

Class	Gains	Freq.	Phases	TIMV*	TSI ACF	WSS	ME	AE
I	Det.	Det.	Det.	—	—	—	—	—
II	Det.	Det.	Rand.	Yes	Yes	Yes	Yes	Yes
III	Det.	Rand.	Det.	No/Yes$^{c \text{ or } d}$	No/Yes^{a}	No/Yes$^{a \text{ and } (c \text{ or } d)}$	No/Yes$^{c \text{ or } d}$	No
IV	Det.	Rand.	Rand.	Yes	Yes	Yes	Yes	No
V	Rand.	Det.	Det.	No/Yes^{b}	No/Yes^{b}	No/Yes^{b}	No/Yes^{b}	No
VI	Rand.	Det.	Rand.	Yes	Yes	Yes	Yes	No
VII	Rand.	Rand.	Det.	No/Yes$^{b \text{ or } c}$	No/Yes^{b}	No/Yes^{b}	No/Yes$^{b \text{ or } c}$	No
VIII	Rand.	Rand.	Rand.	Yes	Yes	Yes	Yes	No

* The acronym TIMV stands for time independent mean value.

^{a} If the boundary condition $\sum_{n=1}^{N}\sum_{m=1,m\neq n}^{N}\exp\{j(\theta_m-\theta_n)\}=0$ is satisfied.

^{b} If the mean value of the random gains is equal to zero.

^{c} If the number of cisoids N is even and $\theta_n=-\theta_{n+N/2}=\pi/2$ for $n=1,\ldots,N/2$.

^{d} If the number of cisoids N is even, the phases θ_n are equal to each other, and the gains are given such that $c_n=-c_{n+N/2}$ for $n=1,\ldots,N/2$.

4. Ertel, R.B., Reed, J.H.: Generation of two equal power correlated Rayleigh fading envelopes. IEEE Commun. Lett. 2(10), 276–278 (Oct 1998)
5. Gutiérrez, C.A.: Channel Simulation Models for Mobile Broadband Communication Systems, Doctoral Dissertations at the University of Agder 16. University of Agder, Kristiansand, Norway (2009)
6. Gutiérrez, C.A., Meléndez, A., Sandoval, A., Rodriguez, H.: On the autocorrelation ergodic properties of sum-of-cisoids Rayleigh fading channel simulators. In: Proc. 2011 European Wireless Conference (EW'2011). pp. 1–6. Vienna, Austria (2011)
7. Gutiérrez, C.A., Pätzold, M.: Sum-of-sinusoids-based simulation of flat-fading wireless propagation channels under non-isotropic scattering conditions. In: Proc. 50th IEEE Global Communications Conference (Globecom 2007). pp. 3842–3846. Washington, DC (Nov 2007)
8. Gutiérrez, C.A., Pätzold, M.: The design of sum-of-cisoids Rayleigh fading channel simulators assuming non-isotropic scattering conditions. IEEE Trans. Wireless Commun. 9(4), 1308–1314 (Apr 2010)
9. Hogstad, B.O., Pätzold, M.: On the stationarity of sum-of-cisoids-based mobile fading channel simulators. In: Proc. 67th IEEE Veh. Technol. Conf. (VTC2008-spring). pp. 400–404. Singapore (May 2008)
10. Leon-Garcia, A.: Probability and Random Processes for Electrical Engineering. Addison-Wesley, New York, second edn. (1994)
11. Parsons, J.D.: The Mobile Radio Propagation Channel. John Wiley and Sons, Chichester, England, second edn. (2000)
12. Pätzold, M.: On the stationarity and ergodicity of fading channel simulators based on Rice's sum-of-sinusoids. Int. Journal of Wireless Information Networks 11(2), 63–69 (Apr 2004)
13. Pätzold, M., Hogstad, B.O.: Classes of sum-of-sinusoids Rayleigh fading channel simulators and their stationary and ergodic properties — Part II. WSEAS Transactions on Mathematics 4(4), 441–449 (Oct 2005)

14. Pätzold, M., Hogstad, B.O.: Classes of sum-of-sinusoids Rayleigh fading channel simulators and their stationary and ergodic properties — Part I. WSEAS Transactions on Mathematics 5(2), 222–230 (Feb 2006)
15. Pätzold, M., Hogstad, B.O., Youssef, N.: Modeling, analysis, and simulation of MIMO mobile-to-mobile fading channels. IEEE Trans. Wireless Commun. 7(2), 510–520 (Feb 2008)
16. Sklar, B.: Digital Communications: Fundamentals and Applications. Prentice Hall, New Jersey, second edn. (2001)
17. Smith, J.I.: A computer generated multipath fading simulation for mobile radio. IEEE Trans. Veh. Technol. 24(3), 39–40 (Aug 1975)
18. Sterian, C.E.D., Ma, Y., Pätzold, M., Banica, I., He, H.: New super-orthogonal space-time trellis codes using differential M-PSK for noncoherent mobile communication systems with two transmit antennas. Annals of Telecommunications (Jul 2010), DOI: 0.1007/s12243-010-0191-1
19. Yip, K.W., Ng, T.S.: Karhunen-Loève expansion of the WSSUS channel output and its application to efficient simulation. IEEE J. Sel. Areas Commun. 15(4), 640–646 (May 1997)
20. Zajić, A.G., Stüber, G.L.: Space-time correlated mobile-to-mobile channels: Modelling and simulation. IEEE Trans. Veh. Technol. 57(2), 715–726 (Mar 2008)
21. Zhang, H., Yuan, D., Pätzold, M., Wu, Y., Nguyen, V.D.: A novel wideband space-time channel simulator based on the geometrical one-ring model with applications in MIMO-OFDM systems. Wirel. Commun. Mob. Comput. 10(6), 758–771 (Jun 2010)